Proposal to Extend Frequency Domain Analysis in VHDL-AMS

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Abstract—Small signal frequency analysis is one of the first standard methods electrical engineers are taught. Despite its limitations it is applied to characterize and design electrical systems up to today. The actual VHDL-AMS standard supports frequency domain analysis based on linearized DAE systems. This makes it difficult to use pure frequency domain characterizations of subcircuits if, for instance, only frequency domain results are required. This paper describes a small extension of the VHDL-AMS standard that helps to overcome the problem if consistency of time and frequency domain descriptions is not required.

Keywords: VHDL-AMS, frequency domain models, small frequency domain analysis, modeling, simulation

I. INTRODUCTION

Charles Proteus Steinmetz formulated the mathematical theory that made the development of alternating current techniques possible [1]. Phasor analysis of linear networks with sinusoid excitation has been widely used and enhanced by electrical engineers since then [2, 3].

Known as AC or frequency domain analysis, the method is also supported by network analysis tools such as Spice and Spectre [4] and behavioral modeling languages such as VHDL-AMS and Verilog-AMS [5, 6]. With respect to the analog part of a system, the languages provide an engineering way to establish the differential algebraic equations (DAE’s) that characterize the analog behavior. Without loss of generality these DAE systems can be written as

\begin{equation}
F(x, x', p, t) = 0
\end{equation}

with \( F : R^n \times R^n \times P \times T \rightarrow R^n \)

\( x : T = [0, \infty) \subset R \rightarrow R^n \)

\( x' \) is the differentiated waveform of \( x \) und \( p \in P \) a vector of parameters. Kirchhoff’s Laws and the constitutive relations that describe the subsystems are taken into consideration.

The basic idea of small signal frequency domain analysis is to linearize the time-domain equations and derive a linear complex system of equations for frequency analysis:

\begin{equation}
\frac{\partial F}{\partial x}\bigg|_{t=0} \cdot X(j\omega) + \frac{\partial F}{\partial x}\bigg|_{t=0} \cdot j\omega \cdot X(j\omega) = RS(j\omega)
\end{equation}

\( X(j\omega) \in C^n \) is the vector of the small signal representations of the \( x \) waveforms at the circular frequency \( \omega = 2\pi \cdot f \) with the frequency \( f \in [0, \infty) \). The vector \( RS(j\omega) \in C^n \) depends on the characteristic of the flow and across sources in the frequency domain.

Whereas the translation from time to frequency domain models is easy to handle, it is much more difficult to derive consistent time models from frequency domain descriptions in the general case. Nevertheless, there exist several problems where only a frequency domain characteristic is given. A typical case is the description of the terminal behavior by frequency-dependent parameters – given for instance by a Touchstone file [7]. A standard approach using such a description in Spice-like simulation engines is to translate the frequency domain descriptions into linear lumped element circuits or state space equations.

The latter procedure is not simple and often not accurate. Frequency domain characteristics are often based on measurements – for instance with a spectrum analyzer. The measurement results are only available in a limited frequency range. It is not reasonable to apply the figured out translation if only a frequency domain analysis is required. Therefore, it makes sense to extend the possibilities of behavioral modeling languages to provide models that are only used for frequency analysis and avoid the translation from frequency to time domain models.

We show in this paper how, with little VHDL-AMS standard extensions, this can be done.

II. CURRENT VHDL-AMS APPROACH

The different equations that build up the system (1) are given by so-called characteristic equations in VHDL-AMS [5]. The description of the constitutive relations of the subsystems can be carried out using simultaneous statements. Access to the current time \( t \) during time-domain simulation is supported by the real-valued function NOW.
Access to the current frequency $f$ during frequency domain analysis is supported by the real-valued function \texttt{FREQUENCY}. It is not allowed to use \texttt{FREQUENCY} in simultaneous statements with respect to the current standard. Thus, the matrices in (2) do not depend on the frequency. \texttt{FREQUENCY} can be used to describe flow and across sources with the help of spectrum quantity declarations. These declarations influence the right-hand side $RS(j\omega)$ of the complex linear system (2) of equations.

\texttt{VHDL-AMS} provides a \texttt{DOMAIN} signal that can be evaluated in the model descriptions. During operation point analysis at $t = 0$ its value is \texttt{QUIESCENT\_DOMAIN}. The value during time-domain analysis is \texttt{TIME\_DOMAIN} and during small signal frequency domain analysis \texttt{FREQUENCY\_DOMAIN}. If the \texttt{DOMAIN} signal is used to establish the simultaneous statements different sets of equations (1) can be established in the different simulations.

\begin{align*}
F_{\text{QUIESCENT\_DOMAIN}}(x,x',p,t) &= 0 \quad (3) \\
F_{\text{TIME\_DOMAIN}}(x,x',p,t) &= 0 \quad (4) \\
F_{\text{FREQUENCY\_DOMAIN}}(x,x',p,t) &= 0 \quad (5)
\end{align*}

Only if (3), (4) and (5) describe the same behavior at time $t = 0$, time and frequency domain models are in accordance.

That means, that the current status of the standard already allows for the description of inconsistent models. This can be demonstrated by the following simple example:

```vhdl-ams
library IEEE;
use IEEE.ELECTRICAL\_SYSTEMS.all;

entity L\_EXAMPLE is
generic (V0 : REAL := 1.0;
            LT : REAL := 1.0;
            LF : REAL := 10.0);
port (terminal P : ELECTRICAL;
      terminal N : ELECTRICAL);
end entity BENCH\_AC;

architecture A0 of L\_EXAMPLE is
quantity V across I through P to N;
begnin
  if \texttt{DOMAIN} = \texttt{QUIESCENT\_DOMAIN} use
    \texttt{V} == \texttt{V0};
  elsif \texttt{DOMAIN} = \texttt{TIME\_DOMAIN} use
    \texttt{V} == \texttt{LT} * \texttt{I'}\texttt{DOT};
  else
    \texttt{V} == \texttt{LF} * \texttt{I'}\texttt{DOT};
  end use;
end architecture A0;
```

Figure 1. Legal \texttt{VHDL-AMS} code example.

This legal \texttt{VHDL-AMS} code (Fig. 1) describes for operating point analysis a voltage source with value \texttt{V0}. For time domain analysis an inductance with value \texttt{LT} is described. For frequency analysis an inductance with value \texttt{LF} is modeled where \texttt{LT} and \texttt{LF} can differ.

III. PROPOSAL FOR EXTENSIONS

A. General Approach

As shown in the previous section special models for frequency domain analysis can be created. However, it is currently not possible to describe a frequency dependent inductance that would allow simultaneous statements such as

$$V = LF(FREQUENCY) * I'\text{DOT};$$

for \texttt{FREQUENCY\_DOMAIN} analysis. Nevertheless, the fixed value \texttt{LF} used in Fig. 1 may describe correctly the frequency behavior at least at one frequency point. Thus, a recall of the simulator with different constant values of \texttt{LF} could realize the relation (6). Certainly, this is not convenient.

Simultaneous statements only establish relations in the time domain. If we want to include frequency dependent features into the existing solutions in a smooth way, we have to establish characteristics in accordance with (5) that describe the frequency behavior in the required way.

Let us discuss a simple example how to construct an associated equation (5) for a given frequency domain behavior. Let $X(j\omega), X2(j\omega) \in C$ be quantities in the frequency domain and $Y(j\omega) \in C$ a frequency dependent coefficient with an imaginary part with $Y.M(\cdot 0) = 0$. The relation between these values is given by

$$X(j\omega) = Y(j\omega) \cdot X2(j\omega)$$

This complex-valued equation cannot be used directly. Left and right hand side expressions in simultaneous statements must be real-valued expressions. The treatment of complex expressions as left and right hand side expressions would require a deep intervention into the existing standard and existing implementations. Using $Y(j\omega) = Y.RE(j\omega) + j \cdot Y.IM(j\omega)$ with $Y.RE(j\omega), Y.IM(j\omega) \in R$ the equation (7) can be rewritten as shown below. It follows for $\omega = 0$

$$X(j\omega) = Y.RE(j\omega) \cdot X2(j\omega)$$

and for $\omega \neq 0$

$$X(j\omega) = Y.RE(j\omega) \cdot X2(j\omega) + \frac{Y.IM(j\omega)}{\omega} \cdot j\omega \cdot X2(j\omega)$$

(8.2)
This characteristic can be derived from the following time-domain representation by linearization as demonstrated in the first section

\[ x_l = Y.RE(j \omega) \cdot x_2 \]  \hspace{1cm} (9.1)

for \( \omega = 0 \) and

\[ x_l = Y.RE(0) \cdot x_2 + \frac{Y.IM(0)}{\omega} \cdot x_2' \]  \hspace{1cm} (9.2)

Applying this procedure to our inductance example the starting point is

\[ V(j \omega) = \frac{j \omega \cdot L F(FREQUENCY) \cdot I(j \omega)}{j \omega \cdot L F(FREQUENCY)} \cdot I(j \omega) \]  \hspace{1cm} (10)

and (6) follows immediately.

Note that, simultaneous statements constructed in this way are (only) valid for frequency domain analysis.

\section*{B. Proposed Standard Extension}

In order to be able to apply the given procedure, the VHDL-AMS standard should allow for the use of the real-valued function FREQUENCY in simultaneous statements. The evaluation of the statements can be done in the following way where the DAE similar to (1) has to be evaluated

\[ F(x, p, t, f) = 0 \]  \hspace{1cm} (11)

with frequency \( f \).

1) Operating point analysis: The domain signal is set to QUIESCENT_DOMAIN. The value of time \( t \) delivered by the function NOW is 0. The value of frequency \( f \) delivered by the function FREQUENCY is 0. The following system extended by initial conditions has to be solved

\[ F(x_0, x_0', p, 0, 0) = 0 \]  \hspace{1cm} (12)

2) Time domain analysis: The domain signal is set to TIME_DOMAIN. The function NOW delivers the current simulation time. The value of frequency \( f \) delivered by the function FREQUENCY is 0. The following DAE system has to be solved

\[ F(x, x', p, t, 0) = 0 \]  \hspace{1cm} (13)

3) Frequency domain analysis: The domain signal is set to FREQUENCY_DOMAIN. The function FREQUENCY delivers the current simulation frequency \( f \). The value of the time delivered by the function NOW is 0. The system for the frequency analysis at \( \omega = 2\pi \cdot f \) is given by the linearized form

\[ \frac{\partial \tilde{F}}{\partial x}(\arg) \cdot X(j \tilde{\omega}) + \frac{\partial \tilde{F}}{\partial \omega}(\arg) \cdot j \tilde{\omega} \cdot X(j \tilde{\omega}) = RS(j \tilde{\omega}) \]  \hspace{1cm} (14)

at \( \arg = (x_0, x_0', 0, \tilde{f}) \). The linearization is carried out around the \( x_0, x_0' \) QUIESCENT_DOMAIN operating point as described in the sections 7.6 and 12.6.5.4 of [5].

All other parts of the standard are not affected. Some commercial VHDL-AMS simulators allow for the usage of the function FREQUENCY. However, it is not documented how it works. To assure the exchange of models between different simulation environment a standard solution is preferred that fulfills the requirement of more general purpose frequency domain features as currently supported by the standard.

\section*{C. Applications}

Forms like (7) can be expressed applying the proposed extensions. Sums of the right-hand products used in (7) can be built up as follows

\[ X1(j \omega) = YA(0) \cdot X2(j \omega) + YB(0) \cdot X3(j \omega) \]  \hspace{1cm} (15)

with \( YA(0) = 0 \) and \( YB(0) = 0 \)

shall be used for frequency domain simulation. Thus, the essential part of the simultaneous statements is given by

```vhdl
-- simultaneous statements
if DOMAIN = FREQUENCY_DOMAIN use
  if FREQUENCY = 0.0 use
    X1 == YA.RE(FREQUENCY)*X2
       + YB.RE(FREQUENCY)*X3;
  else
    X1 ==
      YA.RE(FREQUENCY)*X2
      + YA.IM(FREQUENCY)/MATH_2_PI/FREQUENCY*X2'DOT
      + YB.RE(FREQUENCY)*X3
      + YB.IM(FREQUENCY)/MATH_2_PI/FREQUENCY*X3'DOT;
  end use;
end use;
```

\section*{IV. EXAMPLE - S11 SCATTERING PARAMETERS}

A high frequency impedance with branch voltage \( V(j \omega) \) and branch current \( I(j \omega) \) can be characterized by \( S11(j \omega) \) scattering parameters

\[ B(j \omega) = S11(j \omega) \cdot A(j \omega) \]  \hspace{1cm} (16)
with

\[ A(j\omega) = \frac{1}{2\sqrt{R_0}}(V(j\omega) + R_0 \cdot I(j\omega)) \]  
\[ B(j\omega) = \frac{1}{2\sqrt{R_0}}(V(j\omega) - R_0 \cdot I(j\omega)) \]  

where \( R_0 \) is a reference resistance. This behavior can now be expressed as shown in the previous section. Parameters of the model \( Z_{S11} \) (Fig. 2) are the arrays \( F \) with frequency values, \( S11_{RE} \) with associated real parts of the \( S11 \) parameters, and \( S11_{IM} \) with the associated imaginary parts of the \( S11 \) parameters. The function \( \text{LOOKUP}_1D \) is used to interpolate between the given values [9].

We can transform the \( S11 \) description into an impedance. Another approach is to use (16) to (18) directly and make use in this way of the VHDL-AMS possibilities. \( A \) and \( B \) (multiplied by \( \sqrt{R_0} \)) are declared as free quantities \( AS \) and \( BS \), respectively.

The following architecture \( A0\_AC \) describes the behavior only for frequency domain simulation. It does not work in the time domain.

```vhdl
library IEEE, FUNDAMENTALS_VDA;
use IEEE.ELECTRICAL_SYSTEMS.all;
use IEEE.MATH_REAL.all;
use FUNDAMENTALS_VDA.TLU_VDA.all;

entity Z_S11_AC is
generic (
    F       : REAL_VECTOR;
    S11_RE  : REAL_VECTOR;
    S11_IM  : REAL_VECTOR;
    R0      : REAL := 50.0);
port (
    terminal N1 : ELECTRICAL;
    terminal N2 : ELECTRICAL);
end entity Z_S11_AC;

architecture A0_AC of Z_S11_AC is
quantity V across I through N1 to N2;
quantity AS, BS : REAL;
begins
assert DOMAIN /= TIME_DOMAIN report  
    "ERROR: Time domain model not available."
    severity ERROR;
AS == V + R0*I;
BS == V - R0*I;
if FREQUENCY = 0.0 use
    BS == LOOKUP_1D (FREQUENCY, F, S11_RE) * AS +
      LOOKUP_1D (FREQUENCY, F, S11_IM)
      /MATH_2_PI/FREQUENCY * AS’DOT;
end use;
end architecture A0_AC;
```

Figure 2. Proposed VHDL-AMS code example (S11).

V. CONCLUSION

Although small signal frequency analysis is widely used, establishing models only for frequency domain simulation based on complex frequency dependent relations is only partly supported in the known standardized languages.

The simulation algorithms to evaluate complex frequency domain models are available in the standard simulation tools. Thus, some commercial tools provide methods for more complex frequency descriptions. However to the best of the authors’ knowledge these approaches are not standardized and not documented.

The paper proposes a simple way to include more complex frequency characteristics in a simple way into VHDL-AMS models. The proposal is a compromise with respect to a smooth extension of the language. It should be easy to incorporate it into existing implementations. In a similar way as proposed for VHDL-AMS, the procedure should be applicable to Verilog-A(AMS) [6].

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REFERENCES