We assume that the pair of single ended ports has common reference and is normalized to the same impedance value. We'll give relationships between the vectors and matrices for a port pair, participating in standard-to mixed mode transformation.

For convenience, we will use the two matrices:

$M = \begin{bmatrix} \gamma & -\gamma \\ \gamma & \gamma \end{bmatrix}$ and $K = \begin{bmatrix} 1/\gamma & 0 \\ 0 & \gamma \end{bmatrix}$, where $\gamma = \frac{1}{\sqrt{2}}$.

The inverse of the matrix M is also its transpose: $M^{-1} = M^t$.

<u>S-parameters</u>

The incident and reflected waves in standard and mixed mode are related as follows:

$$A_{mm} = \begin{bmatrix} a_{D1,2} \\ a_{C1,2} \end{bmatrix} = M \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = MA_{std} \qquad \text{(S1)}$$

$$B_{mm} = \begin{bmatrix} b_{D1,2} \\ b_{C1,2} \end{bmatrix} = M \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = MB_{std} \qquad \text{(S2)}$$

$$B_{std} = S_{std} A_{std} \qquad \text{(S3)}$$

$$B_{mm} = S_{mm} A_{mm} \qquad \text{(S4)}$$

By substituting (S1), (S2) into (S4), it is possible to find the relations between the mixed mode and standard mode S-matrices:

$$S_{std} = M^t S_{mm} M \qquad \text{(S5)}$$

$$S_{mm} = M S_{std} M^t \qquad \text{(S6)}$$

<u>Y and Z-parameters</u>

First, we express mixed mode voltages and currents (vectors) via standard mode vectors.

$$V_{mm} = \begin{bmatrix} v_{D1,2} \\ v_{C1,2} \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 1/\gamma & 0 \\ 0 & \gamma \end{bmatrix} \begin{bmatrix} \gamma & -\gamma \\ \gamma & \gamma \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = KMV_{std} \qquad \text{(YZ1)}$$

$$I_{mm} = \begin{bmatrix} i_{D1,2} \\ i_{C1,2} \end{bmatrix} = \begin{bmatrix} 0.5 & -0.5 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} \gamma & 0 \\ 0 & 1/\gamma \end{bmatrix} \begin{bmatrix} \gamma & -\gamma \\ \gamma & \gamma \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = K^{-1} MI_{std} \qquad \text{(YZ2)}$$

Then, define relationships between voltages and currents, in standard and mixed mode.

$$I_{std} = Y_{std} V_{std} \qquad \text{(YZ3)}$$

$$V_{std} = Z_{std} I_{std} \qquad \text{(YZ4)}$$

$$I_{mm} = Y_{mm} V_{mm} \qquad \text{(YZ5)}$$

$V_{mm} = Z_{mm} I_{mm}$ (YZ6)

By substituting (YZ1), (YZ2) into (YZ3-6) we can find relationships between the standard and mixed mode Y and Z-matrices:

$$Y_{mm} = (K^{-1}M) Y_{std} (K^{-1}M)^t \quad \text{(YZ7)}$$

$$Y_{std} = (KM)^t Y_{mm} (KM) \quad \text{(YZ8)}$$

$$Z_{mm} = (KM) Z_{std} (KM)^t \quad \text{(YZ9)}$$

$$Z_{std} = (K^{-1}M)^t Z_{mm} (K^{-1}M) \quad \text{(YZ10)}$$

1. A. Ferrero, M. Pirola. Generalized mixed mode S-parameters, IEEE Trans. on Microwave theory and Techniques, v.54, No.1, 2006.
2. D. Bockelman, W. Eisenstadt. Pure mode network analyzer for on-wafer measurements of mixed mode S-parameters of differential circuits, IEEE Trans. on Microwave theory and Techniques, v.45, No.7, 1997.