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Transformations between STD and MM matrices for a pair of ports

We assume that the pair of single ended ports has common reference and is normalized to the same impedance value. We'll give relationships between the vectors and matrices for a port pair, participating in standard-to mixed mode transformation.

For convenience, we will use the two matrices:

$$M = \begin{bmatrix} \gamma & -\gamma \\ \gamma & \gamma \end{bmatrix} \text{ and } K = \begin{bmatrix} 1/\gamma & 0 \\ 0 & \gamma \end{bmatrix}, \text{ where } \gamma = \frac{1}{\sqrt{2}}.$$

The inverse of the matrix M is also its transpose: $M^{-1} = M^t$.

S-parameters

The incident and reflected waves in standard and mixed mode are related as follows:

$$A_{mm} = \begin{bmatrix} a_{D1,2} \\ a_{C1,2} \end{bmatrix} = M \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = MA_{std} \qquad \text{(S1)}$$

$$B_{mm} = \begin{bmatrix} b_{D1,2} \\ b_{C1,2} \end{bmatrix} = M \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = MB_{std} \qquad \text{(S2)}$$

$$B_{std} = S_{std} A_{std} \qquad \text{(S3)}$$

$$B_{mm} = S_{mm} A_{mm} \qquad \text{(S4)}$$

By substituting (S1), (S2) into (S4), it is possible to find the relations between the mixed mode and standard mode S-matrices:

$$S_{std} = M^t S_{mm} M \qquad \text{(S5)}$$

$$S_{mm} = M S_{std} M^t \qquad \text{(S6)}$$

Y and Z-parameters

First, we express mixed mode voltages and currents (vectors) via standard mode vectors.

$$V_{mm} = \begin{bmatrix} v_{D1,2} \\ v_{C1,2} \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 1/\gamma & 0 \\ 0 & \gamma \end{bmatrix} \begin{bmatrix} \gamma & -\gamma \\ \gamma & \gamma \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = KMV_{std} \qquad \text{(YZ1)}$$

$$I_{mm} = \begin{bmatrix} i_{D1,2} \\ i_{C1,2} \end{bmatrix} = \begin{bmatrix} 0.5 & -0.5 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} \gamma & 0 \\ 0 & 1/\gamma \end{bmatrix} \begin{bmatrix} \gamma & -\gamma \\ \gamma & \gamma \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = K^{-1} MI_{std} \qquad \text{(YZ2)}$$

Then, define relationships between voltages and currents, in standard and mixed mode.

$$I_{std} = Y_{std} V_{std} \qquad \text{(YZ3)}$$

$V_{std} = Z_{std} I_{std}$ (YZ4)

$I_{mm} = Y_{mm} V_{mm}$ (YZ5)

$V_{mm} = Z_{mm} I_{mm}$ (YZ6)

By substituting (YZ1), (YZ2) into (YZ3-6) we can find relationships between the standard and mixed mode Y and Z-matrices:

$Y_{mm} = (K^{-1}M) Y_{std} (K^{-1}M)^t$ (YZ7)

$Y_{std} = (KM)^t Y_{mm} (KM)$ (YZ8)

$Z_{mm} = (KM) Z_{std} (KM)^t$ (YZ9)

$Z_{std} = (K^{-1}M)^t Z_{mm} (K^{-1}M)$ (YZ10)

Mutual STD/MM transformations in case of several port pairs

All above relationships apply only to individual port pairs participating in MM to STD transformations. In case of many such pairs, and also in presence of single ended components together with mixed mode, we need more general description. The transformation matrix in this case will consist of several 2x2 diagonal blocks - each one for every mixed mode port pair - and a unit matrix diagonal block that corresponds to single ended ports.

For example, the combined mixed mode vector X_{mm_blk} and single ended component vector X_{std} could be related as:

$$X_{mm\_blk} = \begin{bmatrix} X_{D1,2} \\ X_{C1,2} \\ X_{D3,4} \\ X_{C3,4} \\ X_5 \end{bmatrix} = \begin{bmatrix} T_{X2} & 0 & 0 \\ 0 & T_{X2} & 0 \\ 0 & 0 & E_1 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix} = T_X X_{std\_blk} \qquad \text{(P1)}$$

Here, T_{X2} is an elementary 2x2 transformation matrix block. Depending on the type of variables in X (incident or reflected wave, voltage or current), it is:

$T_{A2} = T_{B2} = M$ (for incident and reflected wave) (P2)

$T_{V2} = KM$ (for port voltage vector) (P3)

$T_{I2} = K^{-1}M$ (for port current vector) (P4)

E_1 is 1x1 identity matrix. The size of it is defined by the number of individual single ended ports in the left side 'extended' mixed model vector.

Note that (P1) only describes the case when variables on the left and right are 'properly' ordered. The proposed standard allows arbitrary ordering of components in the mixed mode vector. In addition to the block-wise transformation, as shown in (P1), we may consider a chain of several transformations, including permutations and MM/STD transformations:

$$X_{mm} \Leftarrow perm\_mm \Rightarrow X_{mm\_blk} \Leftarrow MM\_to\_STD \Rightarrow X_{std\_blk} \Leftarrow perm\_std \Rightarrow X_{std}$$

For example, possible transformations are:

$$\begin{bmatrix} X_{D3,2} \\ X_{D5,4} \\ X_1 \\ X_{C3,2} \\ X_{C5,4} \end{bmatrix} \Leftrightarrow \begin{bmatrix} X_{D3,2} \\ X_{C3,2} \\ X_{D5,4} \\ X_{C5,4} \\ X_1 \end{bmatrix} \Leftrightarrow \begin{bmatrix} X_3 \\ X_2 \\ X_5 \\ X_4 \\ X_1 \end{bmatrix} \Leftrightarrow \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \end{bmatrix}. \tag{P5}$$

Given a vector X_{mm} (on the left) with components ordered as defined in [Mixed mode order], we need first to permute those components so as to make the mixed mode pairs and block of single ended components (see X_{mm_blk}, next vector). Then, we perform MM-to-STD transformation by way of (P1) and get the respectively grouped vector X_{std_blk}. Finally, we permute single ended vector so as to get the components in X_{std} properly ordered.

Note that two permutations (with mixed mode and standard mode components) need to be independent and therefore require two permutation matrices, P_{std} and P_{mm} respectively. The chain of transformations then becomes:

$$X_{std\_blk} = P_{std} X_{std} \tag{P6}$$

$$X_{mm\_blk} = T_X X_{std\_blk} \tag{P7}$$

$$X_{mm} = P_{mm} X_{mm\_blk} \tag{P8}$$

For example, the matrices used in (P5) are:

$$P_{std} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix}, \; T_X = \begin{bmatrix} T_{X2} & 0 & 0 \\ 0 & T_{X2} & 0 \\ 0 & 0 & E_1 \end{bmatrix}, P_{mm} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}.$$

From (P3-P6) we have:

$$X_{mm} = (P_{mm} T_X P_{std}) X_{std} \tag{P9}$$

$$X_{std} = (P_{std}^T T_X^{-1} P_{mm}^T) X_{mm} . \qquad \text{(P10)}$$

For each type of matrix (*S*, *Y* or *Z*) we have a certain type of input and output vectors. Let us assume that *X* is an input vector (*A*, *V* or *I* respectively) and *U* is an output vector (respectively, *B*, *I* or *V*). Then, similar to (P9) and (P10) there exist relationships for the output vector:

$$U_{mm} = (P_{mm} T_U P_{std}) U_{std} \qquad \text{(P11)}$$

$$U_{std} = (P_{std}^T T_U^{-1} P_{mm}^T) U_{mm} . \qquad \text{(P12)}$$

Note, that permutation matrices are the same (since the components in the input/output vectors must be identically ordered) but the block-wise transformation matrix could be different. As follows from (P3), (P4), in case of *Y* or *Z*-parameters, we need to use different elementary transformation blocks for voltage and current vectors.

In addition to general type input and output vectors (general for S/Y/Z cases), we may define a general transformation matrices, F_{std} and F_{mm} (where *F* stands for *S*, *Y* or *Z*):

$$U_{std} = F_{std} X_{std} \qquad \text{(P13)}$$

$$U_{mm} = F_{mm} X_{mm} . \qquad \text{(P14)}$$

By substituting (P9), (P11) into (P14), we find:

$$F_{std} = (P_{std}^T T_U^{-1} P_{mm}^T) F_{mm} (P_{mm} T_X P_{std}) \qquad \text{(P15)}$$

$$F_{mm} = (P_{mm} T_U P_{std}) F_{mm} (P_{std}^T T_X^{-1} P_{mm}^T) . \qquad \text{(P16)}$$

(P15) shows how the mixed mode matrix, arbitrarily ordered, possibly with single ended terms, can be converted directly into properly ordered single ended form. (P16) shows the reverse transformation.

For example, if the vectors shown in (P5) correspond to incident and reflected waves, and the matrix type is '*S*' then the elementary transformation blocks are those of (P2), and therefore: $T_X = \begin{bmatrix} \gamma & -\gamma & 0 & 0 & 0 \\ \gamma & \gamma & 0 & 0 & 0 \\ 0 & 0 & \gamma & -\gamma & 0 \\ 0 & 0 & \gamma & \gamma & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$. Combining this matrix with permutation matrices shown above, we reduce (P15) into:

$$S_{std} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ -\gamma & 0 & 0 & \gamma & 0 \\ \gamma & 0 & 0 & \gamma & 0 \\ 0 & -\gamma & 0 & 0 & \gamma \\ 0 & \gamma & 0 & 0 & \gamma \end{bmatrix} S_{mm} \begin{bmatrix} 0 & -\gamma & \gamma & 0 & 0 \\ 0 & 0 & 0 & -\gamma & \gamma \\ 1 & 0 & 0 & 0 & 0 \\ 0 & \gamma & \gamma & 0 & 0 \\ 0 & 0 & 0 & \gamma & \gamma \end{bmatrix}.$$

1. A. Ferrero, M. Pirola. Generalized mixed mode S-parameters, IEEE Trans. on Microwave theory and Techniques, v.54, No.1, 2006.
2. D. Bockelman, W. Eisenstadt. Pure mode network analyzer for on-wafer measurements of mixed mode S-parameters of differential circuits, IEEE Trans. on Microwave theory and Techniques, v.45, No.7, 1997.